UNSTEADY MHD MIXED CONVECTION FLOW ADJACENT TO A VERTICAL FLAT SURFACE EMBEDDED IN A THERMALLY STRATIFIED MEDIUM

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ABSTRACT:

The effect of thermal stratification on unsteady mixed convection flow and heat transfer of an incompressible electrically conducting fluid adjacent to a vertical flat surface has been studied in the presence of a transverse magnetic field. The nonlinear governing equations and their associated boundary conditions are initially changed into dimensionless forms by pseudo - similarity variables. The resulting system of equations is then solved numerically using the Keller-box method. The numerical results obtained are compared for steady case with the previously published results and are found to be in good agreement. Further in this study, the effect of stratification parameter and unsteady parameter on local skin friction and heat transfer coefficients in the regime of buoyancy assisting flow are discussed. An analysis of the results obtained shows that the flow field is influenced appreciably by unsteady parameter and thermal stratification parameter. Also, it is found that thermal stratification $(S = 0.25, 0.5)$ causes considerable undershoot in the temperature profiles.

Keywords: Unsteady, MHD, Mixed convection, Stratified medium, Skin friction, Heat transfer

1. INTRODUCTION

The study of buoyancy induced boundary layer flows of Newtonian fluid has gained much attention from researchers because of its application in engineering and industrial fields. The analysis of mixed convection boundary layer flow along a vertical plate has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation for Newtonian fluids have been undertaken by several authors. Ramachandran et al. [1], studied mixed convection flow towards a vertical surface considering both cases of arbitrary wall temperature and arbitrary surface heat flux variations. Other related works can be found in the studies by Ali and Al-Yousef [2] Lin and Hoh [3], Chen [4], Lok et. al., [5] and Anuar Ishak et. al [6].

In fact, thermally stratified flows are of great interest nowadays, stratification of the medium may arise due to a temperature variation, which gives rise to a density variation in the medium. The natural convection flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated by several authors [7-11]. Ishak et.al [12] has obtained the similarity solutions for mixed convection flow over a vertical surface immersed in a stable stratified medium by assuming the external velocity and surface temperature vary linearly with the distance measured from the leading edge of the surface and the same problem has been extended by Ashwini and Eswara [13] by including the effect of magnetic field. Magnetohydrodynamics (MHD) deals with the study of the motions of electrically conducting fluids and their interactions with magnetic fields. In recent years, several

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researchers [14-16] have focused their attention to the MHD flow problems in a stratified medium due to the significant applications.

The above studies are pertaining to steady case. However, in practical situations the flow may be unsteady. Since no attempt has been made to analyze the effect of thermal stratification on the unsteady mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium in the presence of a transverse magnetic field, it is considered in this paper. The external velocity and the surface temperature assumed to vary linearly with the distance measured from the leading edge of the surface. The Keller-box method given in Cebeci and Bradshaw [17] is employed to solve the nonlinear system of this particular problem. The effects of unsteadiness and thermal stratification parameters are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2. GOVERNING EQUATIONS

Let us consider the problem of unsteady mixed convection boundary layer flow over a heated vertical flat surface of temperature $T_w(x)$, which is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$, where $T_w(x) > T_\infty(x)$. It is assumed that $T_w(x) = T_0 + bx$, $T_\infty(x) = T_0 + cx$, and the velocity outside the boundary layer is of the form $U(x) = ax$, where , $a\ b\ ,c$ are constants (with $a > 0$, $b > 0$ and $c \geq 0$) and T_0 is the ambient temperature at the leading edge of the plate. The unsteadiness in the flow field is introduced by the free stream velocity u_e. A magnetic field of strength $B(x)$ is normal to the surface and it is assumed that magnetic Reynolds number of the flow is small enough so that the induced magnetic field is negligible. Under the above assumptions, the equations of continuity, momentum and energy under the boundary layer approximations governing the mixed convection flow on the vertical flat plate can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}(u - u_e) \pm g\beta(T - T_\infty)$$

(2)

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

(3)

Boundary conditions are

$$u = 0,\quad v = 0,\quad T = T_w(x) \quad \text{at} \quad y = 0$$

$$u \to u_e(x, t^*),\quad T \to T_\infty(x) \quad \text{as} \quad y \to \infty$$

(4)

The last term in Eq. (2) represents the influence of the thermal buoyancy force on the flow field with "+" and "-"signs correspond to the assisting and opposing flows, respectively.

We, now introduce the following similarity variables:

$$\eta=\left(\frac{U}{\nu x}\right)^{1/2}(1-\lambda t^*)^{-1/2}y,\quad \psi=(U\nu x)^{1/2}(1-\lambda t^*)^{-1/2}f(\eta,t^*),\quad G(\eta,t^*)=\frac{T-T_\infty}{T_w-T_0},$$

$$M=\frac{\sigma B^2}{\rho U};\quad U=ax;\quad u_e=U(1-\lambda t^*)^{-1};\ t^*=at$$

(5)

where ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. The introduction of the stream function automatically satisfies the continuity Eq. (1). Substituting Eq. (5) into Eqs. (2) and (3) gives

$$f'''+ff''-\frac{\lambda\eta}{2}f''+\lambda(1-f')+1-f'^2+\Omega G-M\left(f'-1\right)=0$$

(6)

$$\Pr^{-1}G''+fG'-\frac{\lambda\eta}{2}G'-f'(S+G)=0$$

(7)

where

$$u=u_e f';\ v=-(1-\lambda t^*)^{-1/2}\left\{\left(\frac{\nu x}{U}\right)^{1/2}\frac{af}{2}+\left(\frac{U\nu}{x}\right)^{1/2}\frac{f}{2}\right\}$$

and primes denote differentiation with respect to η, $\Pr=\nu/\alpha$ is the Prandtl number and $\Omega=\pm Gr_x/\mathrm{Re}_x^2$ [with "$\pm$" sign has the same meaning as in Eq. (2)] is the buoyancy or mixed convection parameter. Further, $Gr_x=g\beta(T_w-T_0)x^3/\nu^2$ and $\mathrm{Re}_x=Ux(1-\lambda t^*)/\nu$ are the local Grashof number and the local Reynolds number, respectively. We notice that Ω is independent of x, with $\Omega=+Gr_x/\mathrm{Re}_x^2>0$ and $\Omega=-Gr_x/\mathrm{Re}_x^2<0$ correspond to the assisting and opposing flows, respectively, while $\Omega=0$ represents the pure forced convection flow. However, we have restricted the present study in the regime of buoyancy assisting flows $(\Omega>0)$. Also, λ is the dimensionless parameter which characterizes the unsteadiness in the flow field. Assuming the value zero to λ, the problem reduces to the steady case. For the nonzero value of λ, the flow is accelerating if $\lambda>0$, provided $\lambda t^*<1$ and, the flow is decelerating if $\lambda<0$.

The boundary conditions (4) now become

$$f(0)=0,\quad f'(0)=0,\ G(0)=1-S,$$
$$f'(\eta)\to 1,\ G(\eta)\to 0\ \text{ as }\ \eta\to\infty,$$

(8)

Where $S=c/b$ is the constant stratification parameter. We notice that $S>0$ implies a stably stratified environment, while $S=0$ corresponds to an unstratified environment.

The primary objective of this study is to estimate the parameters of engineering interest in fluid flow, namely the skin friction and heat transfer coefficients. The dimensionless local skin friction and heat transfer in the form of Nusselt number are given by

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$$C_f(\mathrm{Re}_x)^{1/2} = 2f''(0) \quad \text{and} \quad Nu_x(\mathrm{Re}_x)^{-1/2} = -G'(0)$$

(9)

It is worth mentioning here that when λ = 0 the Eqns. (6) and (7) become

$$f''' + f f'' + 1 - f'^{\,2} + \Omega G - M(f' - 1) = 0$$

$$\mathrm{Pr}^{-1} G'' + f G' - f'(S + G) = 0$$

(10)

which have been considered by Ashwini and Eswara [13], representing the steady counterpart (λ = 0) of the present investigation.

3. RESULTS AND DISCUSSION

The governing equations (6) and (7) with boundary conditions (8) have been solved numerically using implicit finite difference method known as Kellar-Box method. To assess the accuracy of our method, we have compared the steady state present results of skin friction and heat transfer parameter for different Prandtl numbers (0.7 and 7.0) with those of Ramachandran et al. [1] in the absence of stratification parameter (i.e. $S = 0.0$) and magnetic field (i.e $M = 0$) for $\Omega = 1.0$ in Table1. Further, when $M \neq 0$ the steady case $(\lambda = 0)$ results of skin friction and heat transfer coefficients (Fig. 2) are compared with those of Ashwini and Eswara [13]. It is shown that these two results are found to be in excellent agreement.

Pr	Present Results		Ramachandra et. al	
	$F'(0)$	$-G'(0)$	$F'(0)$	$-G'(0)$
0.7	1.7064	0.7645	1.7060	0.7641
7.0	1.5177	1.7251	1.5178	1.7223

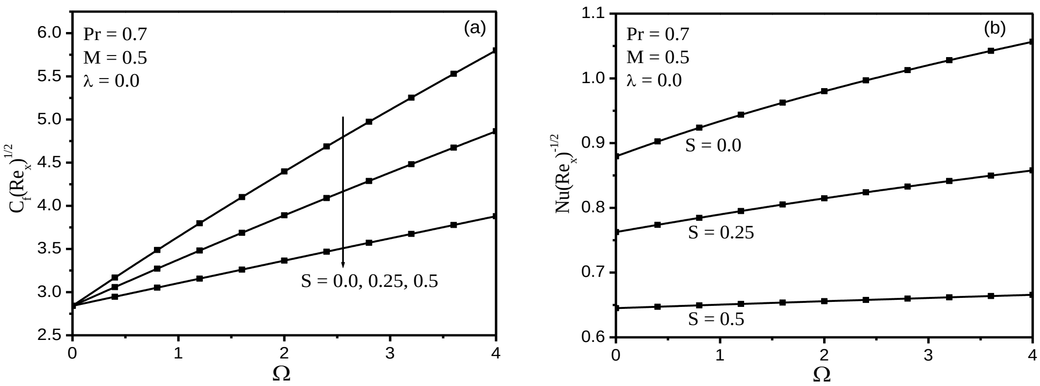

Fig.2 Comparison of steady state (λ = 0) results of (a) skin friction and (b) heat transfer coefficients when

$M \neq 0$ with those of Ashwini and Eswara[13] .

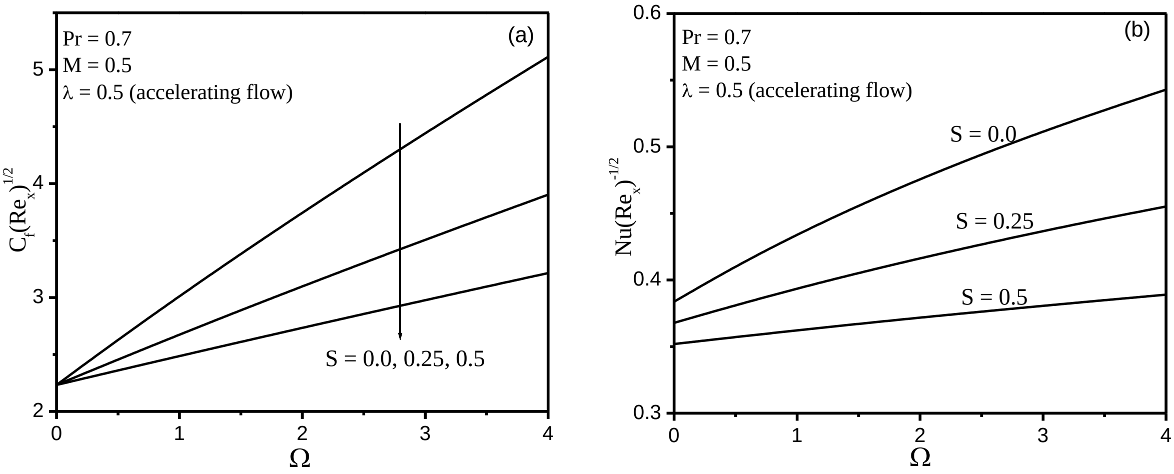

Fig.3 The effect of stratification parameter (S) on (a) skin friction and (b) heat transfer coefficients

when $M = 0.5$ during accelerating flow ($\lambda > 0$)

Fig. 3 display the effect of stratification parameter (S) on local skin friction coefficient [$C_f(\mathrm{Re}_x)^{1/2}$] and heat transfer coefficient [$Nu(\mathrm{Re}_x)^{-1/2}$] during accelerating flow ($\lambda > 0$) with the magnetic parameter ($M = 0.5$) in the regime of buoyancy assisting flows ($\Omega > 0$). It is evident from these figures that both $C_f(\mathrm{Re}_x)^{1/2}$ [Fig. 3(a)] and $Nu(\mathrm{Re}_x)^{-1/2}$ [Fig. 3(b)] decrease as S increases. This is due to the fact that an increase in the thermal stratification parameter decreases the buoyancy force due to the thermal gradients and retards the flow. Consequently, the thermal boundary layer thickness increases and thus decreases the heat transfer rate between the fluid and wall. To be more specific, the percentage of decrease in skin friction and heat transfer coefficients is about 10.8% and 5.65% at $\Omega = 3.0$.

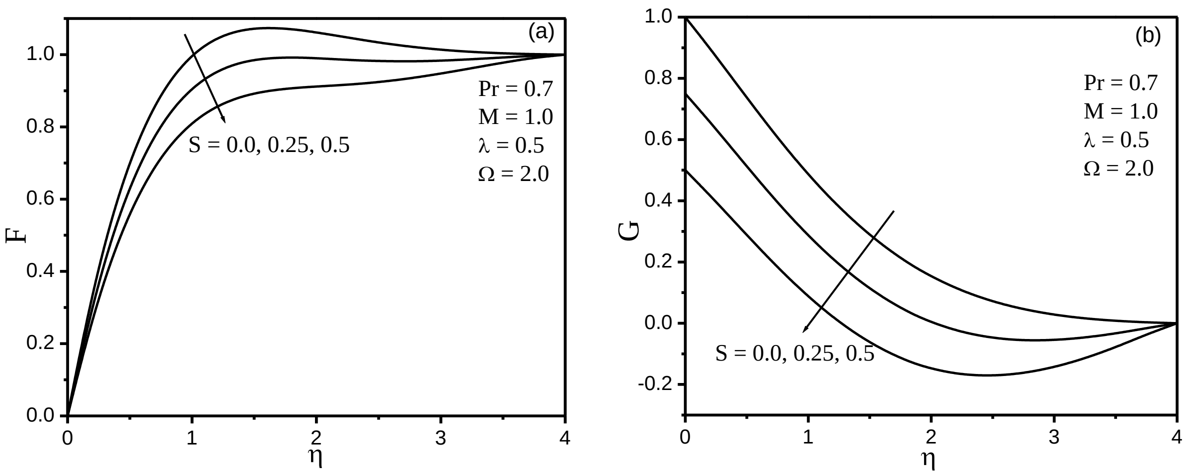

Fig.4 The effect of stratification parameter (S) on (a) velocity and (b) temperature profiles

when $M = 1.0$ during accelerating flow ($\lambda > 0$)

Under the influence of magnetic field (M), the stratification parameter (S) on velocity (F) and temperature (G) profiles in the case of accelerating flow ($\lambda > 0$) and buoyancy assisting flow ($\Omega > 0$) are presented in Fig. 4. It is observed that both velocity and temperature

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decrease, near the surface, with the increase in stratification parameter. Also, there is an undershoot in the temperature profiles (G), under the influence of stratification parameter $(S = 0.25, 0.5)$ confirming the heat transfer reversal from fluid to surface (instead of surface to fluid).

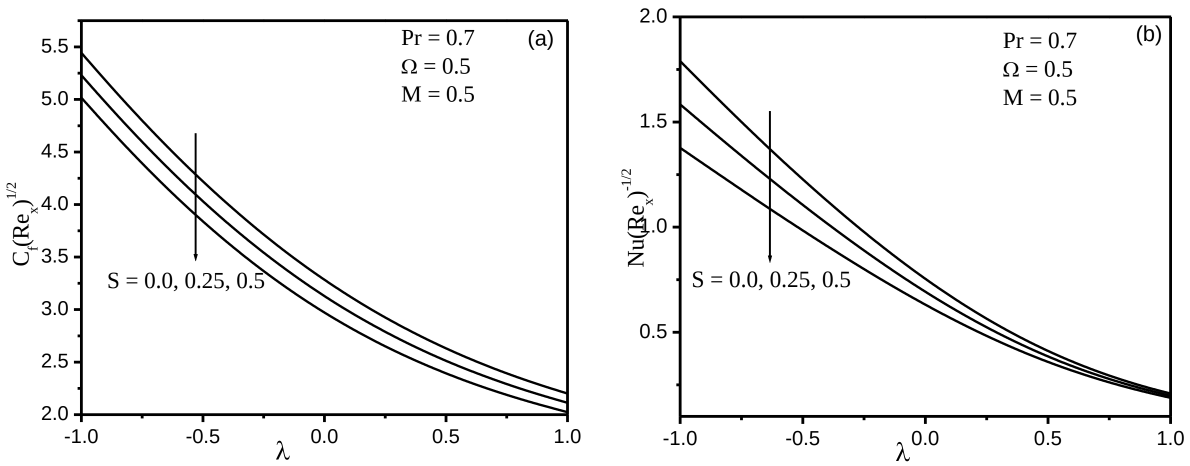

Fig.5 The effect of stratification parameter (S) on (a) skin friction and (b) heat transfer coefficients

with the unsteady parameter (λ) when $M = 0.5$ and $\Omega = 0.5$

The effect of thermal stratification (S) with the unsteady parameter $(-1.0 \leq \lambda \leq 1.0)$ on skin friction $[C_f(\mathrm{Re}_x)^{1/2}]$ and heat transfer $[Nu(\mathrm{Re}_x)^{-1/2}]$ coefficients under the influence of magnetic field $M = 0.5$ and when $\Omega = 0.5$ is exhibited in Fig.5. It is observed that $C_f(\mathrm{Re}_x)^{1/2}$ and $Nu(\mathrm{Re}_x)^{-1/2}$ decreases with the increase of thermal stratification. Further, it is found that $C_f(\mathrm{Re}_x)^{1/2}$ decreases gradually as unsteady parameter increases from decelerating flow $(\lambda < 0)$ to accelerating flow $(\lambda > 0)$. On the other hand, $Nu(\mathrm{Re}_x)^{-1/2}$ decreases with the increase of unsteady parameter and reaches zero when $\lambda \geq 1.0$. In fact, the percentage of decrease in $C_f(\mathrm{Re}_x)^{1/2}$ at $\lambda = -0.9$ is about 42% and $Nu(\mathrm{Re}_x)^{-1/2}$ is about 37.51%.

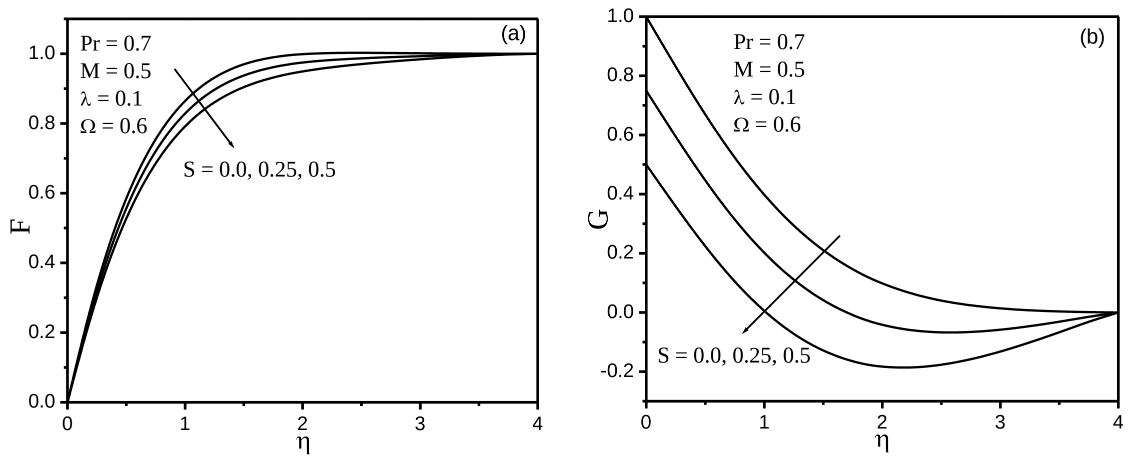

Fig.6 The effect of stratification parameter (S) on (a) velocity and (b) temperature profiles when

$\boldsymbol{M = 0.5}$, $\boldsymbol{\lambda = 0.1}$ ***and*** $\boldsymbol{\Omega = 0.5}$

Fig.6 shows the effect of thermal stratification parameter (S) on velocity (F) and temperature (G) profiles in the buoyancy assisting flow ($\Omega = 0.6$) when $\lambda = 0.1$(accelerating flow) and $M = 0.5$. It is clear from the figure that both velocity and temperature is found to decrease, near the surface, as stratification parameter increases. As a result, momentum boundary layer thickness increases whereas thermal boundary layer thickness decreases. Further, the thermal stratification parameter ($S = 0.25, 0.5$) causes considerable undershoot in the temperature profiles (G). This confirms that heat transfer from fluid to surface instead of surface to fluid.

4. CONCLUSIONS

This work considered the unsteady, MHD mixed convection boundary layer flow adjacent to a vertical flat surface in a stable thermally stratified medium. The results show that thermal stratification, along with the unsteady parameter in the buoyancy-assisted flow regime, leads to a reduction in both skin friction and heat transfer coefficients. Consequently, the momentum boundary layer becomes thicker, while the thermal boundary layer becomes thinner. The presence of thermal stratification also causes an undershoot in the temperature profiles, indicating a temperature defect in the flow.

Nomenclature

		Greek symbols	
a,b,c	constants	σ	electrical conductivity
f	dimensionless stream function	ρ	density
F	dimensionless velocity	α	thermal diffusivity
G	dimensionless temperature	β	thermal expansion coefficient
g	acceleration due to gravity	η	similarity variable
Gr_x	local Grashof number	λ	buoyancy or mixed convection parameter
$\Pr$	Prandtl number	ν	kinematic viscosity
Re_x	local Reynolds number	ψ	dimensional stream function
S	Stratification parameter		
T	fluid temperature		
B	Strength of uniform magnetic field	***Subscripts***	
M	dimensionless magnetic parameter	w	condition at the wall
T_0	ambient temperature at the leading	∞	condition away from the wall
	edge		
$T_w(x)$	surface temperature	***Superscripts***	
$T_\infty(x)$	ambient temperature	'	Differentiation with respect to η
u, v	velocity components along the x and y directions, respectively		
$U(x)$	velocity outside the boundary layer		
x, y	Cartesian coordinates along the surface and normal to it, respectively		

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REFERENCES

1. Ramachandran, N., Chen, T.S and Armaly, B.F.: Mixed Convection in stagnation flows adjacent to vertical surfaces. Journal of Heat Transfer, **110**, no. 2, pp. 373-377, 1988.
2. Ali, M. and Al-Yousef, F.: Laminar mixed convection from a continuously moving vertical surface with suction or injection. *Heat and Mass Transfer*, **33**, no. 4, pp. 301-306, 1998.
3. Lin, H. T. and Hoh, H. L.: Mixed convection from an isothermal vertical flat plate moving in parallel or reversely to a free stream. *Heat and Mass Transfer*, **32**, no. 7, pp. 441-445, 1997.
4. Chen, C. H.: Mixed convection cooling of a heated, continuously stretching surface. *Heat and Mass Transfer*, **36**, no. 1, pp. 79-86, 2000.
5. Lok, Y. Y., Amin, N., Campean, D. and Pop, I.: Steady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. *International Journal of Numerical Methods for Heat and Fluid Flow*, **15**, no. 7, pp. 654-670, 2005.
6. Ishak, A.: Boundary layer flow adjacent to a vertical plate with constant surface temperature. *Sains Malaysiana*, **39**, no. 6, pp. 1035-1039, 2010.
7. Chen, C. and Lin, C.: Natural convection from an isothermal vertical surface embedded in a thermally stratified high-porosity medium. *Int. J. Eng. Sci.*, **33**, pp. 131-138, 1995.
8. Tanny, J. and Cohen, J.: The mean temperature field of a buoyancy induced boundary layer adjacent to a vertical plate immersed in a stratified medium. *Int. J. Heat Mass Transfer*, **41**, pp. 2125-2130, 1998.
9. Takhar, H. S., Chamkha, A. J. and Nath, G.: Natural convection flow from a continuously moving vertical surface immersed in a thermally stratified medium. *Heat and Mass Transfer*, **38**, pp. 17-24, 2001.
10. Saha, S. C. and Hossain, M. A.: Natural convection flow with combined buoyancy effects due to thermal and mass diffusion in a thermally stratified media. *Nonlinear Analysis: Modelling and Control*, **9**, No. 1, pp. 89-102, 2004.
11. Saha, S. C., Gu, Y. T., Md. Molla, M., Sadia Siddiq and Hossain, M. A.: Natural convection from a vertical plate embedded in a stratified medium with uniform heat source. *Desalination and Water Treatment*, **44**, pp. 7-14, 2012.
12. Ishak, A., Nazar, R. and Pop, I.: Mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium. *International Journal of Heat and Mass Transfer*, **51**, pp. 3693-3695, 2008.
13. Ashwini, G., Poornim, C. and Eswara, A. T.: Hydromagnetic mixed convection flow adjacent to a vertical flat plate immersed in a thermally stratified medium. *National Conference on Applied and Engineering Mathematics*, p.35, 2011.

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Peer Reviewed & Refereed Journal (IF: 7.9)
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14. Chamkha, A. J.: Laminar hydromagnetic natural convection flow along a heated vertical surface in a stratified environment with internal heat absorption. *Can. J. Phys.*, **80**, pp. 1145-1156, 2002.

15. Gurminder Singh, Sharma, P. R. and Chamkha, A. J.: Effect of thermally stratified ambient fluid on MHD effective flow along a moving non-isothermal vertical plate. *International Journal of Physical Sciences*, **5** (3), pp. 208-215, 2010.

16. Mukhopadhyay, S.: MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. Alexandria Engineering Journal, **52**, pp. 259-265, 2013.

17. Cebeci, T. and Bradshaw, P.: Physical and computational aspects of convective heat transfer, Springer, NewYork, 1984.