CREEP RESPONSE AND STRUCTURAL OPTIMIZATION OF FUNCTIONALLY GRADED ROTATING DISKS

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ABSTRACT

Rotating disks are widely used in high-speed engineering applications such as gas turbines, turbo generators, compressors, flywheels, automotive braking systems, and aerospace components, where they are frequently subjected to elevated temperatures and severe mechanical loading. Under such operating conditions, creep deformation and thermo-mechanical stresses significantly influence the structural integrity, reliability, and service life of the rotating components. The present study focuses on the creep deformation and stress analysis of rotating disks made of composite and functionally graded materials (FGMs). A comprehensive review of elastic-plastic stress analysis, steady-state creep behavior, and thermo-mechanical performance of rotating disks has been carried out by considering various material models, thickness profiles, and reinforcement distributions. Particular emphasis has been given to aluminum matrix composites reinforced with silicon carbide particles or whiskers and their functionally graded counterparts operating under thermal gradients. The study examines the influence of material anisotropy, reinforcement content, thermal residual stresses, and grading profiles on stress distribution and creep strain rates in rotating disks. The findings reveal that functionally graded and composite rotating disks exhibit superior creep resistance and improved stress distribution compared to homogeneous disks. The investigation provides valuable guidelines for the optimal design and development of advanced rotating disks capable of operating efficiently under high-temperature and high-speed conditions.

Keywords: Rotating disks, creep deformation, thermo-mechanical stress analysis, functionally graded materials (FGMs)

1.0 INTRODUCTION

Rotating disks are among the most important structural components used extensively in rotating machinery such as steam and gas turbines, turbo generators, pumps, compressors, flywheels, automotive braking systems, ship propellers, and computer disk drives (Gupta et al., 2005; You et al., 2007; Hojjati and Hassani, 2008). Owing to their wide range of engineering applications, rotating disks continue to attract significant research interest in the field of thermo-mechanical analysis and material design. In many practical applications, these disks operate under severe working conditions involving high rotational speeds and elevated temperatures. Such operating environments generate substantial thermal and mechanical stresses within the disk material (Farshi and Bidabadi, 2008). Consequently, the material experiences creep deformation, which adversely affects the structural integrity, efficiency, and service life of the rotating components (Laskaj et al., 1999; Farshi et al., 2004; Gupta et al., 2005, 2007). For example, in turbine rotors, heat transfer from the external surface to the shaft and bearings may significantly reduce the operational efficiency and reliability of the system (Bayat et al., 2008).

The increasing use of composite materials and functionally graded materials (FGMs) in rotating disks operating under high-temperature and high-stress conditions necessitates a comprehensive understanding of their creep characteristics and deformation mechanisms. Knowledge of creep strain rates and stress distribution is essential for predicting the useful life and long-term performance of rotating disks under specified loading conditions. Such information also enables engineers to optimize processing parameters, modify material composition, and improve disk geometry to achieve enhanced creep resistance and structural reliability. Since creep deformation directly influences the durability and efficiency of rotating machinery, designers are continuously interested in controlling creep behavior to extend service life and improve operational performance. Therefore, creep analysis plays an important role in optimizing the distribution of reinforcement in composite rotating disks and in minimizing the adverse effects of thermo-mechanical loading.

Although considerable research has been carried out on functionally graded materials, the development of a unified design framework for selecting suitable material combinations and composition profiles still remains a challenge for engineers and researchers. The present study aims to contribute toward the understanding of creep behavior in rotating composite and functionally graded disks subjected to severe thermo-mechanical loading conditions. The investigation is expected to provide useful guidelines for the design and optimization of rotating disks made from uniform composites as well as FGMs, thereby contributing to the development of reliable and efficient design methodologies for advanced rotating machinery applications.

2.0 ROTATING DISK

Rotating disk provides an area of research and studies due to their vast utilization in rotating machinery *viz.* steam and gas turbine rotors, turbo generators, pumps, compressors, flywheels, automotive braking systems, ship propellers and computer disk drives (Gupta *et al*, 2005; You *et al*, 2007; Hojjati and Hassani, 2008). In most of these applications, the disk has to operate at elevated temperature and is simultaneously subjected to high stresses originating due to disk rotation at high speed (Farshi and Bidabadi, 2008). As a result of severe mechanical and thermal loadings, the material of the disk undergoes appreciable creep deformations, thereby affecting its performance (Laskaj *et al*, 1999; Farshi *et al*, 2004; Gupta *et al*, 2005). For example, in turbine rotor there is always a possibility that heat from the external surface is transmitted to the shaft and then to the bearings, which may adversely affect the functioning and efficiency of the rotor (Bayat *et al*, 2008).

Optimal and more reliable design of rotating disks has long been an important issue in engineering design. By changing geometrical parameters of the disk and physical properties of the disk material, optimal and more reliable design of rotating disk for given operating conditions (*i.e.* load, speed, operating temperature) can be achieved. The general parameters varied in the optimization process are the geometrical parameters like the mean radius and thickness of the disk, and material properties such as density, elastic modulus and Poisson's ratio (Guven and Celik, 2001).

3.0 ANALYSIS OF ELASTIC-PLASTIC STRESSES IN ROTATING DISK

Analysis pertaining to estimation of stresses and strain rates in thin rotating disk can be found in most of the standard text books and literature (Malkin, 1934; Finnie and Heller, 1959; Lubhan and Felger, 1961; Odqvist, 1974; Findely *et al*, 1976; Kraus, 1980; Boyle and Spance, 1983; Skrzypek and Hetnarski, 1993; Nabarro and Villiers, 1995; Penny and Mariott, 1995). Timoshenko and Goodier (1970) was the first to obtain closed form solutions for rotating homogeneous disks but without considering temperature gradient. Reddy and Srinath (1974) and Chang (1976) investigated the influence of material density on stresses and displacements in a rotating disk made of orthotropic material. It is demonstrated that the existence of density gradient in the disk significantly affects the distribution of stresses and displacements. Zhou and Ogata (2002) obtained closed form solutions for rotating solid disk made of cubic anisotropic material by using direct displacement method. The displacement, strain and stress were expressed as a simple function of polar coordinates. Orcan and Eraslan (2002) obtained analytical solution for elastic-plastic deformation in a variable thickness rotating disk in the form of power function. The analysis assumed the Tresca's yield criterion and its associated flow rule, and linear strain hardening material behavior. The solution obtained was verified by comparing it with the solution available for uniform thickness disk. It is observed that with the reduction in disk thickness the plastic limit angular velocity increases and the magnitudes of stresses and deformations reduce.

In the design and manufacturing of gas turbine rotor, optimization of weight is an important parameter. Several numerical techniques have been proposed for optimizing the profile of rotating disk (Fox, 1970; Zienkiewicz and Campbell, 1973; Malkov and Salganskaya, 1976; Pederson, 1981; Wang and Gallagher, 1985; Vanderplaats, 1990; Cheu, 1990). In turbo jet engines, rotating disk is simultaneously subjected to mechanical and thermal loads. The disk may also be subjected to internal pressure due to shrink fitting on a shaft. In addition, the blade effects may also be modeled by applying external tensile load at the outer radius of the disk.

When the disk rotates at significant angular velocity, while the gases cross through fins, there is a resulting temperature gradient imposed on the disk.

Kollman (1978, 1981, 1984) solved the problem of shrink fitted disk, both rotating as well as non-rotating, using Tresca's yield criterion. Yeh and Han (1994) proposed a symmetric formulation to predict elastic stresses in an inhomogeneous rotating disk with arbitrary thickness and operating under thermal loading.

Gas turbine disks mostly operate under high temperature gradients and is also subjected to high angular velocity. High speed results in large centrifugal forces in the disk and simultaneous presence of high temperature reduces the strength of disk material, which may ultimately result in increased deformation of the disk. In order to attain accurate and reliable analysis of stress distribution in the disk, the solution should consider changes in the material properties caused by temperature change. To achieve this goal, Farshi *et al* (2004) considered an inhomogeneous disk model with variable thickness. Using the variable material properties method (Jahed and Dubey, 1997; Jahed and Sherkatti, 2000; Jahed and Shirazi, 2001), stresses were obtained in a rotating disk operating under steady temperature field. The stress calculation was followed by optimization process based on inscribed hypersphere method. In the optimization process, the objective function was the total weight of the disk and the constraints were imposed on the stresses, which were kept less than the yield stress of the material. The disk profile was optimized and the final solutions were obtained. The study indicates that the solutions of optimization process for different initial profiles of the disk with similar specifications are unique when the inscribed hypersphere radii in the last solution stages are equal. Results obtained were compared with the published data and were found to be in good agreement.

Hasan (2007) carried out analysis of elastic–plastic stresses in a curvilinear orthotropic rotating annular disk by considering strain – hardening material behavior. Radial and circumferential stress components were estimated for different angular velocity of the disk. It is observed that the magnitudes of circumferential stress components are higher than the radial stress components. The magnitudes of residual stress component of the circumferential stress and plastic flow are highest at the inner surface. At all angular velocities, the radial displacements for both elastic and plastic solutions have higher values at the inner surface than observed at the outer surface of the disk.

The concept of functionally graded materials in which the properties vary continuously from point to point, may be considered in order to make optimal use of material and to improve the performance of structural components by suitable control of stresses and deformations along with controlling the undesirable features such as internal residual stress (Nagatha and Takahashi, 1995; Williamson *et al*, 1995). Jain *et al* (1999) demonstrated that a specific radial variation of elasticity modulus leads to equal radial and circumferential stresses in the rotating disk. Horgan and Chan (1999) also reported similar results**.**

Durodola and Adlington (1997) carried out predictive assessment of the effects of various forms of gradation of material properties on the elastic stresses and strains in rotating axisymmetric components such as disks and rotors. The study assumed isotropic FGMs based on particulate metal-matrix composites. Durodola and Attia (2000) carried out similar analysis for rotating hollow and solid disks subjected to centrifugal body load. The disks were assumed to be made of FG orthotropic materials obtained by non-uniform reinforcement of metal-matrix with long fibers. Stresses and displacements in the disks were obtained by direct integration of the governing differential equations and compared with those obtained by using finite element method. A very good agreement is observed between these results.

Kordkheili and Naghdabadi (2007) obtained semi-analytical thermo-elastic solution for hollow and solid rotating axisymmetric disk made of FGM under plane stress condition.

Bayat *et al* (2007) investigated FG rotating disk subjected to axisymmetric bending and steady-state thermal loading. The material properties of the disk were assumed to be graded in the direction of thickness according to a power law distribution of volume fractions of the constituents. Using small deflection theory, an exact solution for displacement field was obtained. It is observed that for particular values of grading index (n) of material properties, the mechanical response of the FG disk can be smaller than that of a homogeneous disk.

Bayat *et al* (2008) obtained elastic solutions for FG axisymmetric rotating disks having variable thickness profile. The material properties and disk thickness profile were assumed to be represented by separate power-law distributions. The effects of material grading index and geometry of the disk were investigated on the stresses and displacements in the disk. It is observed that the maximum radial stress in the solid FG disk with parabolic thickness profile is not at the centre like uniform thickness disk. The study reveals that FG disk with parabolic concave or hyperbolic convergent thickness profile can be more efficient than the uniform thickness disk.

Sharma and Sahni (2009) used Seth's transition theory to obtain elastic-plastic stresses in thin rotating disks made of transversely isotropic and purely isotropic materials. It is observed that the rotating disk made of transversely isotropic material yields at a higher angular speed as compared to disk made of isotropic material. Rotating disk made of transversely isotropic material is observed to be much safer than the disk made of isotropic material.

Asghari and Ghafoori (2010) obtained semi-analytical three-dimensional elasticity solution for FG hollow and solid rotating disks. The study aimed to generalize an available two-dimensional plane-stress solution to a three-dimensional one. It is revealed that for thin disks the two-dimensional solution provides appropriate results but for thick disks a three-dimensional elasticity solution should be considered to avoid poor results.

Sharma and Sahni (2011) used transition theory to obtain elastic – plastic and transitional stresses in a variable thickness rotating disk having inclusion. It is observed that the rotating disk made of incompressible material with inclusion requires higher angular speed to yield at the internal surface as compared to disk made of compressible material. For disk with exponentially varying thickness, high angular speed is required for initial yielding at the internal surface as compared to flat disk. It is also concluded that the disk made of isotropic compressible material is on the safer side of design as compared to disk made of isotropic incompressible material as it requires higher percentage increase in angular speed to become fully plastic from its initial yielding.

4.0 ANALYSIS OF CREEP IN ROTATING DISK

Disks of gas turbines, jet engines, and automotive and aerospace braking systems usually operate at relatively higher angular speed and high temperature or thermal gradient. Therefore, the prediction of creep deformations is extremely important for these applications. Most of the published work on creep in rotating disk is dedicated to the steady state creep behavior since a major part of the component's creep life is spent in the secondary stage.

Wahl *et al* (1954) analyzed steady state creep deformations and stress distributions in a rotating forged disk made of 12% chromium steel at 1000 ^0F. The results obtained were validated experimentally. Theoretical analysis of creep was carried out using von Mises and Tresca yield criteria, while the creep behavior was described by power law. The stress distributions obtained using Tresca and Mises criteria do not differ significantly. The normalized strain curves calculated for various values of stress exponent (n) corresponding to both the yield criteria are observed to practically coincide for n = 6 to 9. The study also reveals that the theoretical and experimental stresses are in better agreement if the Tresca criterion is used. The creep deformations estimated using Mises theory are found to be quite low as compared to those obtained experimentally, which may be attributed to anisotropy of the disk material.

Wahl (1956) derived some formulas for calculating stress distributions in rotating disks having constant and variable thickness, undergoing steady-state creep at elevated temperature. The formulas were based on the Tresca's yield criterion and the associated flow rule and give reasonable results when compared with the available experimental data (Wahl *et al,* 1954). The method proposed was also applied to calculate the transient change in stress when the stress distribution changes from an initial to a steady state condition during the starting period.

Wahl (1957) utilized the formulas derived in their previous work (Wahl *et al,* 1954) to construct the design charts of stress distribution in constant thickness disks, undergoing steady state creep, for different values of stress exponent (n) and diameter ratios. In all the cases, the disks were subjected to a radial peripheral load to simulate the effect of blade loading. The steady state creep rate was expressed as a product of power function of stress multiplied by a function of time.

Ma (1960) extended his work for variable thickness solid disks, used in gas turbine and jet engine, operating at uniform temperature. The study used Tresca's criterion and its associated flow rule while the steady state condition was described by exponential creep law. It is revealed that the stress distributions over central portion of variable thickness disk are quite different from those observed in a constant thickness disk.

Ma (1961; 1964) further extended his analysis for variable thickness disk operating under variable temperature. The steady state creep was described by either exponential creep law (Ma, 1961) or power law (Ma, 1964)**.** The study reveals that the proposed analyses can be used to obtain closed form solutions for complex disk design problem with great simplicity instead of using tedious numerical solutions.

Wahl (1963) investigated the effects of initial transient period on the long- time creep tests of rotating disks by using both time-hardening and strain-hardening relations. The results obtained were applied to the long-time spin-tests conducted on steel disks at 1000 ^{0}F (Wahl *et al,* 1954). The study reveals that by considering the effects of transient period, there is no appreciable impact on the over-all creep deformation noticed during the spin tests. However, when the creep deformations are of order of elastic strains it is necessary to include such transient effects. Arya and Bhatnagar (1979) analyzed the creep stresses and deformations in rotating disk made of orthotropic materials by assuming the creep rate to be a function of time. A numerical example was worked out to investigate the effect of transition (nonsteady) creep and orthotropicity of material on stress and strain distributions in the disk. It is observed that the tangential stress at any radius and the tangential strain at the inner radius of the disk decrease at all times for an anisotropic material. The time taken to reach steady state distribution decreases with increasing anisotropy of the disk material.

Bhatnagar *et al* (1986) carried out analysis of steady state creep in rotating disks having constant and variable (linear and hyperbolic) thickness. The creep stresses and strains were obtained for different cases of anisotropy by using Norton's power law creep model. The study reveals that the selection of a certain type of material anisotropy and an optimum disk profile would lead to a better disk design.

Gupta *et al* (2000) used Seth's transition theory to estimate creep stresses and creep rates in a thin rotating disk having variable thickness and variable density. The study reveals that a rotating disk with radially varying density and thickness ratio is on the safer side of design as compared to a flat disk having variable density.

Singh and Ray (2001; 2003a) analysed steady state creep in a rotating disk made of isotropic FGM (Al-SiC_p) and operating at a constant temperature. The disk was assumed to have linearly decreasing content of SiCp from the inner to outer radius and creeping according to Norton's power law. It is observed that the steady state creep response of the FGM disk in terms of strain rates is significantly superior as compared to a similar disk having uniform distribution of SiCp.

Singh and Ray (2002) studied the influence of anisotropy, induced due to processing, on the steady state creep in a rotating disk made of 6061Al-20 *wt%* SiC_w and operating at 561 *K*. The creep behavior of the disk material was described by Norton's power law. It is observed that the presence of anisotropy in the disk leads to significant reduction in both the tangential and radial strain rates over the entire disk radius.

Jahed and Bidabadi (2003) presented a method to analyze primary and secondary creep in axisymmetric rotating disks and pressure vessels subjected to different types of loading, such as internal and external pressure, centrifugal loading and temperature gradient. The method uses the basic solution for a rotating disk made of uniform isotropic material and generates the solution for disk made of non-uniform inhomogeneous material. Primary and secondary creep was predicted and the results obtained were compared with those estimated by FEM technique. In all the cases, a good agreement is observed.

Singh and Ray (2003b) analyzed steady state creep in a rotating disk made of anisotropic 6061Al-20 *wt%* SiC_w composite in the presence of thermal residual stress. The study used Norton's power law and the newly proposed yield criterion, which at appropriate limits reduces to Hill anisotropic and Hoffman isotropic yield criterion. It is observed that the presence of residual stress in an anisotropic disk results in significant changes in the distribution of tangential stress but similar distribution of radial stress when compared with those obtained in a similar anisotropic disk but without residual stress. The presence of tensile residual stress in the disk leads to significant increase in creep rate as compared to that observed in a similar anisotropic disk without residual stress. In the presence of residual stress, the radial strain rate becomes tensile in the middle of the disk, however, it remains compressive towards the inner and outer radii. Similar results were also noticed in the subsequent study by Singh and Ray (2004), based on the effect of thermal residual stress in a rotating disk made of isotropic 6061Al-20 *vol%* SiC_w and undergoing steady state creep. The study used Hoffman yield criterion for isotropic material.

Gupta *et al* (2004b, 2005) investigated steady state creep in a rotating disk made of isotropic FGM containing SiCp in a matrix of pure aluminum and operating in the presence of radial thermal gradient. The creep behavior of the composite was described by Sherby's law. The study indicates that the steady-state strain rates in the FGM disk are significantly lower than that observed in an isotropic disk having uniform distribution of SiCp, when both the disks operate under thermal gradient. The study also reveals that the strain rates in composite disk operating under thermal gradient are lower as compared to a similar disk operating at a constant temperature, estimated by averaging the imposed thermal gradient.

Gupta *et al* (2007) used Artificial Neural Network (ANN) for predicting the creep response of a rotating Al-SiCp composite disk operating at elevated temperature. The analysis of steady state creep, described by Sherby's law, was carried out for various combinations of SiCp size and content, and operating temperature. The creep parameters were extracted from the limited experimental uniaxial creep data available for Al-SiCp. The results obtained were used to train the ANN model based on back propagation learning algorithm with SiCp size, SiCp content and temperature as the input and stresses and strain rates as the output parameters. The predictions obtained from the ANN model were compared with the corresponding values obtained by analytical procedure. A nice agreement is observed between the creep stresses and strain rates predicted by ANN model and estimated analytically.

Singh (2008) used Norton's creep law to analyze steady state creep in a rotating disk made of 6061Al-20 *vol%* SiC_w with varying extent of anisotropy, characterized by a parameter a. It is observed that the radial strain rate, which remains compressive for isotropic composite disk (a = 1.0) and anisotropic disk having $a > 1$, becomes tensile in the middle of the disk when the extent of anisotropy parameter $a < 1$. By changing the extent of anisotropy from $a > 1$ to $a < 1$, the variation of tangential strain rate in the disk remains similar, however, its magnitude reduces by about five orders. Thus, the presence of anisotropy introduces significant change in the strain rates, though its effect on the resulting stress distribution is relatively small.

Gupta *et al* (2009a) developed a mathematical model to predict the steady state creep response of a rotating disk made of SiC (particle/whisker) reinforced 6061Al matrix composite. The model was used to investigate the effect of SiC morphology on the creep behavior of composite disk. The steady state creep behavior was described by Sherby's creep law. The study reveals that the creep stresses and creep rates in the disk are significantly affected by the morphology of SiC. The steady state creep rates in a whisker reinforced disk are observed to be significantly lower than those observed in a particle reinforced disk.

Gupta *et al* (2009b) formulated a mathematical model to describe steady state creep behavior in rotating disks made of isotropic aluminum matrix composite containing linear and quadratic distributions of SiCp in the radial direction. The disks were assumed to operate under a radial thermal gradient, originating due to braking action as estimated by FEM analysis. The steady state creep behavior of the disks was described by Sherby's law. Based on the developed model, the distributions of stresses and strain rates were obtained and compared for various FGM disks containing the same average amount (20 *vol*%) of SiCp. The study reveals that the creep stresses and steady state creep rates in a rotating FGM disk can be significantly reduced by employing more SiCp reinforcement in the middle of the disk as compared to the inner and the outer radii.

Loghman *et al* (2011) investigated time-dependent redistribution of creep stress in a rotating disk made of Al–SiC composite by using Mendelson's method of successive elastic solution. The material creep behavior was described by Sherby's constitutive model by using available experimental results on Al–SiC composite. It is concluded that the uniform distribution of SiC reinforcement does not have considerable influence on stresses. However, the minimum and the most uniform distribution of circumferential and effective thermoelastic stresses belongs to FGM disk made of aluminum with 0% SiC at the inner surface and 40% SiC at the outer surface. It has also been found that the stresses, displacement and creep strains change with time at a decreasing rate in such a manner that after almost 50 years the solution approaches the steady state condition.

5.0 CONCLUSIONS

a) Rotating disks operating under high temperature and high rotational speed experience significant thermo-mechanical stresses and creep deformation, which strongly influence their structural reliability and service life.

b) Functionally graded materials and composite materials provide improved stress distribution and enhanced creep resistance compared to conventional homogeneous materials.

c) The distribution of reinforcement content, material anisotropy, thermal gradients, and disk geometry significantly affect the creep strain rates and stress profiles in rotating disks.

d) Functionally graded rotating disks with optimized reinforcement distribution exhibit lower creep strain rates and better thermo-mechanical performance under severe operating conditions.

e) The reviewed analytical and numerical studies demonstrate that appropriate material gradation and disk design can effectively improve the efficiency, safety, and durability of rotating machinery components.

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